Long Distance Communication System

NRZ format:
propagate near the zero dispersion point (ZDP)

variation of ZDP by as much as 1 nm
Soliton format:

Standard fiber

\[ D(\lambda) = \text{constant} \]
- energy decreases exponentially
- pulse width increases exponentially

Dispersion decreasing fiber

\[ D(\lambda) \sim e^{-\Gamma z} \]
- energy decreases exponentially
- pulse width remains constant
• need to determine the variation of chromatic dispersion along an optical fiber

• methods proposed recently:
    • estimated from mode-field diameter
    • based on modulated-instability-induced-gain
    • phase-matching condition in four wave mixing
    • four wave mixing
Current Method

- output waveform carries information about the fiber
- determine fiber dispersion from a theoretical model of signal propagation in fiber

Advantage

- can be used to determine other fiber parameters if a suitable model is used
Theory and Algorithm

$q(z=0) \rightarrow f_T(z) \rightarrow q_T(z=L)$ (experiment)

Non-Linear Schrödinger Equation

\[ i \frac{\partial q}{\partial z} + \frac{1}{2} f(z) \frac{\partial^2 q}{\partial t^2} + |q|^2 q = 0 \]

\[ \rightarrow q(z=L) \] (simulation)

1. Guess $f(z)$
2. Calculate $q(z=L)$
3. Compare $q(z=L)$ and $q_T(z=L)$; adjust $f(z)$ accordingly
4. Repeat step 1 through 3 until $f(z) \sim f_T(z)$
How to choose $\Delta f$?

- Define an error function: $E[q(L,t), q_T(L,t)]$

  e.g. 1. $E(q, q_T) = \int |q(L) - q_T(L)|^2 dt$

  2. $E(q, q_T) = \int \left| |q(L)|^2 - |q_T(L)|^2 \right|^2 dt$

- Expand $f(z) = \sum_{\mu=1}^{N} f_\mu \psi_\mu(z)$

  $\psi_\mu$: orthogonal function
\[ \Delta E = \sum_{\mu=1}^{N} \frac{dE}{df_{\mu}} \Delta f_{\mu} \]

Choose \[ \Delta f_{\mu} = -\alpha \frac{dE}{df_{\mu}} \]

\[ \Delta E = -\alpha \sum_{\mu=1}^{N} \left( \frac{dE}{df_{\mu}} \right)^2 < 0 \]

**Gradient Descent!**

\[ \frac{dE}{df_{\mu}} = \int \left( \frac{\delta E}{\delta q} \frac{\partial q}{\partial f_{\mu}} + \frac{\delta E}{\delta q^*} \frac{\partial q^*}{\partial f_{\mu}} \right) dt \]
and \( \frac{\partial q}{\partial f_\mu} \), \( \frac{\partial \hat{q}}{\partial f_\mu} \) satisfy the linearized non-linear Schrödinger equation:

\[
\frac{i}{2} \frac{\partial \varphi}{\partial z} + \frac{1}{2} f(z) \frac{\partial^2 \varphi}{\partial t^2} + 2|q|^2 \varphi + q^2 \varphi^* = -\frac{1}{2} \frac{\partial f}{\partial f_\mu} \frac{\partial^2 q}{\partial t^2}
\]

\( \Delta f \) can be determined from given \( q(z=L) \) and \( q_T(z=L) \)

1. The convergence of gradient descent algorithm is too slow, we use **conjugated gradient** instead.

2. In feasibility test, \( f_T(z) \) is also generated from the non-linear Schrödinger equation.
Results

Without Loss

Target profile:
\[ f_T(z) = \exp[(\ln 0.25) z] \]

Initial guess:
\[ f(z) = 0.2 \]

Input pulse:
\[ \frac{0.5}{\sqrt{\sqrt{\pi}}} e^{\frac{t^2}{2}} \]
With Loss (\(\Gamma=0.69\))

Target profile:
\[ f_T(z) = \exp[(\ln0.25) z] \]

Initial guess:
\[ f(z) = 0.2 \]

Input pulse:
\[ \frac{1.5}{\sqrt{\sqrt{\pi}}} e^{-\frac{t^2}{2}} \]
With Loss and 3rd order dispersion
($\Gamma=0.69$, $\beta=1$)

Target profile:
$f_T(z)=\exp[(\ln0.25)z]$

Initial guess:
$f(z) = 0.2$

Input pulse:
$$\frac{1.5}{\sqrt{\sqrt{\pi}}} e^{-\frac{t^2}{2}}$$
Summary

Reconstruction of the dispersion profile from an input output pulse profile

Further Study

- Effects of timing jitters and amplitude jitters
- Experimental verification