

LES of the Interaction and Partial Reconnection of Unequal Strength Vortices

Louis DUFRESNE

*Mechanical Engineering Department, Division TERM, and
Centre for Systems Engineering and Applied Mechanics (CESAME)
Université catholique de Louvain, Belgium*

Grégoire WINCKELMANS

*Mechanical Engineering Department, Division TERM, and
Centre for Systems Engineering and Applied Mechanics (CESAME)
Université catholique de Louvain, Belgium*

Among the possible means to enhance the decay of wake vortices is the use of aircraft high-lift devices (e.g. the flaps) to produce a multiple vortex system that exhibits fast growing instabilities. These instabilities can in turn lead to strong interactions between the vortices and thus a high three-dimensionalization of the vorticity wake field with an important production of small scale structures. In the end, this can contribute to the dissipation of a significant amount of energy with a resulting much weakened and simpler vortex system. It is in that perspective that we undertook the present investigation on two generic problems of much interest in terms of vortex dynamics and reconnection. The first one is concerned with the unstable dynamics and decay of counter-rotating four-vortex systems. The second one is on the interaction and partial reconnection of two unequal orthogonal vortices.

The numerical results we present here were obtained by solving the conservative formulation of the incompressible Navier-Stokes equations

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) = -\nabla P + \nu \nabla^2 \mathbf{u} - \nu_{\text{hv}} \nabla^8 \mathbf{u}, \quad (1)$$

where $P = p/\rho$ is the reduced pressure, together with the incompressibility constraint

$$\nabla \cdot \mathbf{u} = 0, \quad (2)$$

using a standard Fourier-Galerkin projection in all three Cartesian directions [2]. The continuity equation is satisfied by a reprojection of the velocity field onto a divergence free space. A phase-shift procedure for dealiasing and a 3rd order Runge-Kutta scheme for time integration complete the spectral solver.

The large-eddy simulation (LES) modeling is based on a hyper-viscous (8th order) term which was shown to provide an appropriate separation of scales—sufficient dissipation at the small scales but with negligible effects on the dynamics of the large ones. Comparisons with

other more standard LES models, made on similar vortex interaction problems, confirmed the appropriateness of this choice [3]. For the very high Reynolds number simulations, we used a quasi-Euler formulation which is obtained by setting $\nu = 0$ in equation (1) and relying only on hyper-viscosity for dissipation.

Our first set of results was obtained for the dynamics of four-vortex systems, as sketched in figure 1, and characterized by essentially two non-dimensional parameters: the relative circulation ratio Γ_2/Γ_1 (negative in counter-rotation) and the relative vortex spacing b_2/b_1 . Linear stability analysis has shown that some parametric configurations are very unstable [4]. Here, we focus on two particular cases.

The first one, with $\Gamma_2/\Gamma_1 = -0.37$ and $b_2/b_1 = 0.48$, has previously been studied experimentally [6] at a Reynolds number of $Re = \Gamma_0/\nu = 1.0 \times 10^5$ (with $\Gamma_0 = \Gamma_1 + \Gamma_2$). The simulation of this case was carried out, at the same Reynolds number, using a collocation grid of $720 \times 320 \times 224$ (about 52 million) points. The computational domain was set with a longitudinal extent of $L_x = 11.1 b_1$ to allow for the development of one equivalent Crow long wavelength instability; in the transversal directions, the dimensions were set to $L_y = 4.93 b_1$ and $L_z = 3.45 b_1$. The (Gaussian) vortex core radii were set to $a_1/b_1 = 0.065$ and $a_2/a_1 = 0.54$, as measured experimentally.

From the simulation results it is possible to see that, as the secondary vortices orbit around the primary ones, a series of unstable modes with very different wavelengths develop. There is a short elliptic-like mode ($\lambda_s/b_1 = O(0.1)$), a medium wavelength mode ($\lambda_m/b_1 = O(1)$), and a Crow type long wavelength one ($\lambda_l/b_1 = O(10)$). These instabilities have been triggered by adding a low amplitude white-noise perturbation to the base flow field.

The medium wavelength instability dominates the process and is the one leading to the strong distortion of the secondary (weaker) vortices. These latter deform to a point at which they form loops—the so-called Omega-loops—that wrap around the primary vortices which are only lightly deformed in comparison. A similar phenomenon has also been observed in numerical simulations of unequal anti-parallel vortices with moderate strength differences and at much lower Reynolds numbers [5]. The partial reconnection that occurs between the secondary and primary vortices leads to the formation of a series of periodic circulation variations along the primary vortices. These local variations then propagate in a wave-like manner to eventually collide with each other producing local bursting, as observed in the experiment [6]. From this and the colliding of the remaining parts of the Omega-loops, through the symmetry plane, result the production of a significant amount of small scale structures and thus an important dissipation of energy.

This sequence of events was further put in evidence in our second quasi-Euler simulation with the parametric configuration $\Gamma_2/\Gamma_1 = -0.3$ and $b_2/b_1 = 0.3$ together with $a_1/b_1 = 0.075$ and $a_2/a_1 = 2/3$; the other simulation parameters remained unchanged. The evolution of this second system follows the same pattern as what has just been described above, except that this time the vortex interaction and partial reconnection appear to be much stronger with a more important production of small scale structures. An example of energy characterization is shown in figure 2, it illustrates the evolution of the total (E) and base flow (E_0) energies as well as some 3D energy spectra taken at different times. In the early stages of the (inviscid) dynamics, there is no dissipation. When the instabilities develop and

the flow becomes significantly three-dimensional, the base flow energy begins its decrease while the total energy remains constant. The total energy starts its decay only when the (hyper-) viscous reconnection process begins. We can also see from the 3D spectra that the flow field becomes nearly turbulent with what appears to be a narrow inertial range.

One of the most important factor for the production of small scale structures, and to the significant energy decay associated with it, seems to be governed by the complex dynamics linked to the partial reconnection between the secondary and primary vortices. In order to gain some insight on this, we then considered the more fundamental problem of the interaction of two unequal orthogonal vortices.

For this simulation, we set the initial condition to correspond to the previous quasi-Euler case with a circulation ratio of $\Gamma_2/\Gamma_1 = 0.3$ and also with $a_2/a_1 = 2/3$; the collocation grid had $(256)^3$ points on a computational domain of dimension $(2\pi)^3$. We note that here because of the intrinsic (unsteady) three-dimensionality of the base flow field no initial perturbation is required to initiate the reconnection process.

In the early stages of the interaction, the secondary (weaker) vortex significantly deforms to eventually reconnect (anti-) parallel with the main vortex. As the secondary vortex comes close enough to the primary one small bridges, or fingers, of vorticity are formed. The apparition of these vorticity bridges has also been observed in numerical simulations of equal strength vortex reconnection at lower Reynolds numbers [1]. In the present case though, the reconnection process is only partial and is seen to take place essentially on the periphery of the main vortex. An illustration of the vortices, after reconnection has begun, is shown in figure 3. The circulation “steps” that have hence formed move away from each other to eventually collide with their periodic images. This results again in the production of a significant amount of small scale vorticity structures with its corresponding energy decay.

References

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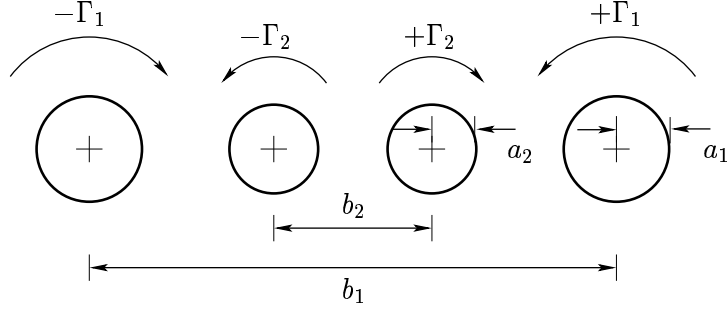


Figure 1: Base flow configuration of a four-vortex system: the primary vortices (subscript 1) have a circulation Γ_1 and a spacing b_1 , the secondary vortices (subscript 2) have Γ_2 and b_2 respectively.

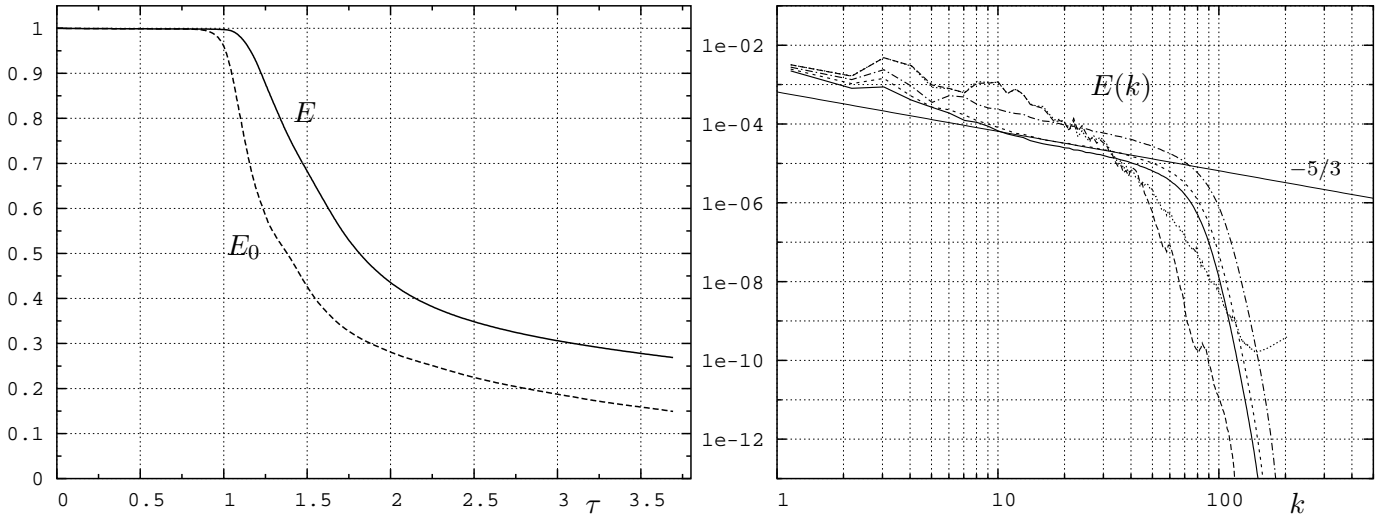


Figure 2: Time evolution of the total (E) and base flow (E_0) energies, on the right, for the four-vortex system with $\Gamma_2/\Gamma_1 = -0.3$ and $b_2/b_1 = 0.3$. On the left, the 3D energy spectra at times $\tau = 0$ (dot), 0.62 (dash), 1.66 (dash-dot), 2.57 (short dash), and 3.69 (solid).

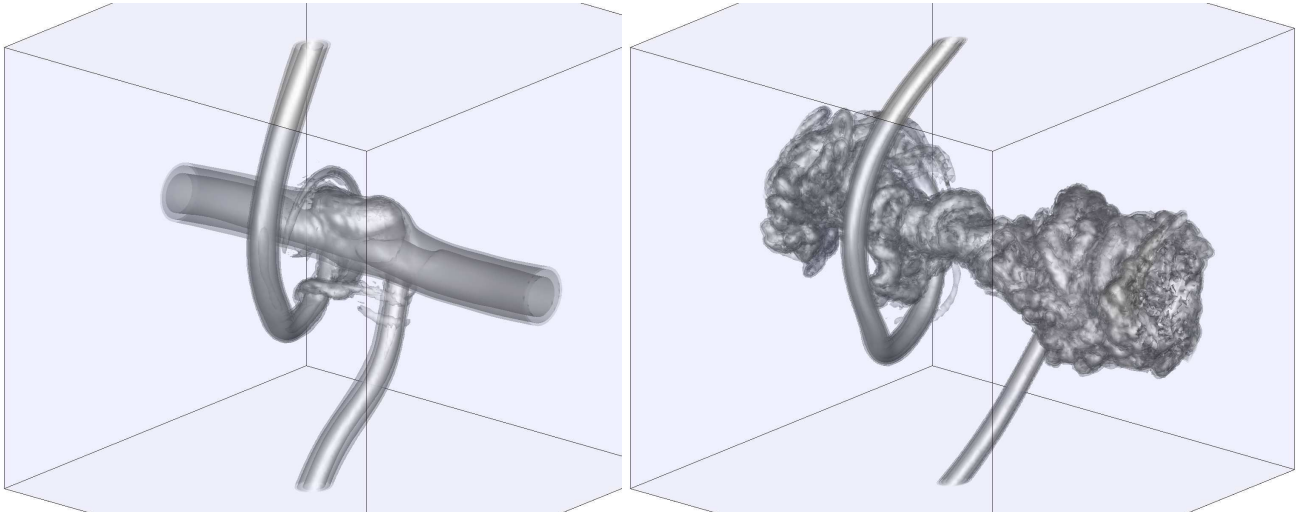


Figure 3: Illustration (vorticity iso-surfaces) of the partial reconnection of two unequal ($\Gamma_2/\Gamma_1 = 0.3$) orthogonal vortices at times $\tau = 19.1$ (left) and 40.5 (right).