High gradient phenomena in two-dimensional vortex interactions

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Hyperdiffusion, a simple linear eddy diffusivity scheme, is commonly used in atmospheric and oceanic simulations because it increases the range of inertially behaving spatial scales for a given model resolution. Compared with molecular diffusion (which is utterly negligible in the atmosphere and oceans), hyperdiffusion more sharply confines the dissipation to the smallest scales of the numerical model. But is this all that hyperdiffusion does? In this paper, the inelastic interaction of two distributed vortices of unequal size is examined. Contour surgery (CS) simulations are compared with pseudospectral (PS) simulations employing hyperdiffusion or molecular diffusion. The example illustrates what is believed to be the most fundamental characteristic of two-dimensional (2-D) and layerwise-2-D vortex dynamics, namely, the formation of exceedingly high vorticity gradients. There is an excellent agreement between the hyperdiffusive PS and CS calculations at early times (i.e., for a few vortex rotation periods). Thereafter, significant discrepancies develop, beginning abruptly from the time when vorticity-gradient intensification is arrested by diffusion. A rapid inward erosion of the smaller of the two vortices then takes place. This erosion takes place under the joint action of (hyper) diffusion and stripping (the peeling of the vortex periphery by the external flow). With hyperdiffusion, the erosion is accompanied by a serious numerical artifact: a climb in the peak vorticity by 30% in this example. Eventually, the erosion reaches the vortex center and the vortex is sheared into a filament. In the CS calculation, there is no erosion, no climb in peak vorticity, and the vortex appears to last indefinitely. In the PS calculations, the viscosity or hyperdiffusion is adjusted according to the resolution to give the largest possible inertial range while ensuring numerical stability. It is found that vortices that are spanned by fewer than 10–20 grid points are eroded away in only a few vortex rotation periods (a time scale that is very much shorter than one would estimate from pure viscous decay). These findings bring into question the results of many 2-D turbulence simulations using hyperdiffusion, for hyperdiffusion simulates neither inviscid dynamics nor molecular-diffusive dynamics. © 1995 American Institute of Physics.

I. INTRODUCTION

Much recent research has been devoted to simulating and quantifying high Reynolds number two-dimensional (2-D) flows as part of an effort to find improvements in modeling atmospheric and oceanic flows. The predictability of flow phenomena is of the essence and is dependent, both on the algorithm used and on the parameters chosen for conducting a simulation. Almost all numerical algorithms in present use are essentially grid based, and an unanswered question is what effect this has on simulating typical high Reynolds number flow phenomena having a characteristic spatial scale at, or well below, the grid scale.

These grid-based or conventional algorithms depend on some sort of eddy-diffusivity scheme to maintain numerical stability. The primary purpose of an eddy-diffusivity scheme is to maximally extend the range of inertially behaving spatial scales for a given model resolution. A secondary purpose is to properly account for the collective effects of the unresolved spatial scales. There is as yet, however, little guidance for what these collective effects should be.

A simple, commonly employed eddy-diffusivity scheme is hyperdiffusion. In this paper, we take a closer look at this scheme by comparing pseudospectral (PS) calculations employing both hyperdiffusion and molecular diffusion with contour surgery (CS) calculations. CS, a practically inviscid numerical method, can explicitly resolve the range of scales being parametrized by hyperdiffusion in PS.

CS is an extension and refinement of the original Lagrangian contour dynamics method for piecewise-uniform vorticity distributions. Like all numerical methods aiming to conduct long-time simulations, CS includes a form of small-scale enstrophy dissipation (called "surgery"). However, surgery is unlike molecular (or hyper)diffusion because it does not affect high transverse vorticity gradients, in particular, those frequently found on the periphery of vortices. Rather, surgery damps fine-scale undulations along vorticity contours, and selectively removes filamentary structures (structures that are arguably quasipassive).

Recently, these two methods were compared at high resolution in the simulation of the stripping of a broadly distributed, initially circular vortex by external adverse shear. For the duration of the calculation (12.5 vortex rotation periods), it was found that the two methods remained in quantitatively close agreement, even when as few as eight
discrete vorticity levels were employed in the CS calculation.

Both numerical methods have been used to study fundamental vortex interactions in 2-D and layerwise 2-D flows. Those studies relevant to the present one include the PS study of distributed, symmetric vortex merger by Melander and Zabusky and the CS study of the inelastic interaction of two, generally unequal-sized vortex patches by Dritschel and Waugh.

In the present study, we examine the inelastic interaction of two unequal-sized distributed vortices. We are primarily interested in the regime where the numerical spatial resolution in PS is low to moderate, as in conventional PS simulations of 2-D turbulence. In this regime, even when hyperdiffusion is employed, vortices spanned by 10–20 grid points initially may be destroyed in only a few vortex rotation periods in PS while surviving apparently indefinitely in CS. Furthermore, there may be a significant (≈30%) temporary vorticity amplification at the center of a vortex when hyperdiffusion is used. We believe that these numerical processes may have contributed to false physical insights drawn from calculations of 2-D turbulence.

The outline of the paper is as follows. In the next section we describe the initial conditions used, and, briefly, the numerical algorithms employed. In Sec. III we summarize the numerical experiments conducted and list various diagnostics central to our quantitative analysis. In Sec. IV we compare the results of the two calculation methods, first pictorially and then by way of the diagnostics. In Sec. IV we also contrast normal (molecular) diffusion and hyperdiffusion in the PS calculations. In Sec. V we offer our conclusions.

II. INITIAL CONDITIONS

We have chosen the initial conditions so that the evolution falls into the “partial straining-out” (PSO) regime of inelastic vortex interactions (IVIs), where the smaller vortex sheds a significant part of its peripheral vorticity into incoherent filamentary debris, while the larger vortex remains nearly unaffected. This PSO regime is arguably an important one in the limit of dilute turbulence, when most interactions may be expected to be grazing ones.

The two vortices have the same initial vorticity profile shape, given by

\[ \omega(r) = \omega_{\text{max}} \left\{ \begin{array}{ll} 1 & (r \leq R_1) \\ 1 - \frac{r-R_1}{R_0-R_1} & (R_1 \leq r \leq R_0) \\ 0 & (R_0 \leq r) \end{array} \right. \]

where

\[ f(x) = \exp \left[ -kx^{-1} \exp \left( \frac{1}{x-1} \right) \right] \]

with constants \( k = 2.560 \times 10^4 \) and \( \omega_{\text{max}} = 2\pi \). We take the fractional width of the vortex edge, \( \delta = (R_0-R_1)/R_0 = 0.5 \), and \( R_0 = 1.0 \) and 0.5 for the larger and smaller vortex, respectively (see Fig. 1). Therefore, initially, the circulation ratio \( \Gamma_{\text{PS}} / \Gamma_0 \) between two vortices is 0.09. The choice of \( \delta \) is a compromise between what could be handled best by each numerical method.

The PS method aims to solve the 2-D vorticity equation, namely

\[ \omega_t + u \omega_x + v \omega_y = \nu v^2 \omega - \nu_4 \nabla^4 \omega, \]

in which \( \nu_2 \) and \( \nu_4 \) are the coefficients of normal (molecular) viscosity and hyperdiffusion, respectively, in a doubly periodic domain of side length \( 2\pi \) (see Refs. 9 or 10 and references therein for additional information on the method).

The CS method aims to solve the inviscid version of (1) in Lagrangian form—see Ref. 4 for details. In CS, for reasons of computational efficiency, doubly periodic boundary conditions are not employed in this study; rather, the flow is simulated on the infinite plane. The principal effect of a doubly periodic geometry is to induce an overall rigid rotation on the flow because it is necessary that the average vorticity be zero in this geometry. Thus, to compare the CS and PS results, we rotate the CS results through the angle \( \Delta \phi = \Gamma_0 t/8\pi^2 \), where \( t \) is the time and \( \Gamma_0 \) is the total initial circulation in the CS experiment. The quantitative analyses below show that the additional effects of doubly periodic geometry are negligible.

III. NUMERICAL EXPERIMENTS

A. Run Information

In PS, the vorticity equation is solved with the aid of FFTs and time splitting (see Ref. 13 for the numerical scheme). The highest resolution used is 512, with hyperdiffusion \( \nu_4 = 1.953 \times 10^{-9} \) or viscosity \( \nu_0 = 1.28 \times 10^{-4} \). Dealiasing is not used because the aliasing error is negligible here.
TABLE I. Vorticity discretization in CS and contours in PS. Note: the last column gives the ratio of the total circulation contained within contour \( j \) divided by the total circulation of the vortex.

<table>
<thead>
<tr>
<th>Contour number</th>
<th>( \omega ) just inside contour in CS</th>
<th>Corresponding ( \omega ) in continuous profile</th>
<th>( \Gamma_j / \Gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.112 815 66</td>
<td>0.059 747 00</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.232 059 90</td>
<td>0.169 509 24</td>
<td>0.9862</td>
</tr>
<tr>
<td>3</td>
<td>0.360 012 36</td>
<td>0.293 107 59</td>
<td>0.9634</td>
</tr>
<tr>
<td>4</td>
<td>0.492 355 54</td>
<td>0.423 245 41</td>
<td>0.9314</td>
</tr>
<tr>
<td>5</td>
<td>0.626 301 14</td>
<td>0.556 389 80</td>
<td>0.8896</td>
</tr>
<tr>
<td>6</td>
<td>0.759 302 64</td>
<td>0.689 873 33</td>
<td>0.8367</td>
</tr>
<tr>
<td>7</td>
<td>0.887 235 60</td>
<td>0.820 340 58</td>
<td>0.7698</td>
</tr>
<tr>
<td>8</td>
<td>0.998 938 11</td>
<td>0.940 158 31</td>
<td>0.6790</td>
</tr>
</tbody>
</table>

We will use the abbreviation “nPS” to denote a calculation performed using normal viscosity, and “hPS” to denote one performed using hyperdiffusion. The viscosity coefficients \( v_2 \) and \( v_4 \) are chosen from the relation

\[ v_2 k_{\text{max}}^2 = v_4 k_{\text{max}}^4 = c = 8.389 \times 1.335 \omega_{\text{max}}, \]

for different resolutions. (Such a choice is in the middle of the range of values commonly employed. If \( c \) is too small, numerical instability develops. To our knowledge, no rigorous numerical stability criterion is available.) We make this choice to get the largest inertial range for a given model resolution, under the constraint of numerical stability.

As for the time integration, a time step of 0.005 is taken for the 64" and 128" runs, 0.0025 for the 192" and 256" runs, and 0.00125 for the 512" runs.

In CS, eight discrete vorticity levels are used to approximate the continuous vorticity profile of each vortex (for the procedure, see Ref. 3, for the vorticity levels, see Table I. Note that the results here are comparable in accuracy with the 16-level case examined in Ref. 9 due to the initial vortex edge used in this study—the reader may wish to consult Ref. 9 to see the high level of accuracy that was obtained in that case). The dimensionless node-spacing parameter \( \mu \) is 0.06 in the highest-resolution CS calculation. This corresponds to a surgical scale, \( \delta \), equal to

\[ \delta = 7 \times 10^{-5} L_B, \]

where \( L_B = 2 \pi \) is the box width. The time step used in all CS calculations is 0.1. Using a smaller time step does not significantly improve the accuracy of the calculations. Table II summarizes all the experiments performed. All calculations were run until \( t = 50 \), with data sampled every 0.2 units of time. By \( t = 50 \), the smaller vortex has completed approximately two rotations around the larger one.

B. Viscometric and diagnostic methods

An effective, sensitive way to determine the quantitative differences between the calculated flow fields is to examine individual vorticity contours, particularly those within the smaller vortex core. Below, we describe the various steps taken to obtain accurate diagnostic results.

1. Identification

The first step is the identification of the vortex cores. One has to be careful, owing to the severe deformation of the smaller vortex at early times. We define the vortex core for the smaller vortex as the region within a high-level vorticity contour, \( \omega_s / \omega_{\text{max}} = 0.556 \) (contour #5 in Table I). We have investigated several other choices for \( \omega_s / \omega_{\text{max}} \) (apart from very low values), and the results we obtained were similar. We have chosen contour #5 because it contains a large portion (89%) of the total circulation of the smaller vortex initially. Any expelled filaments are not considered part of the smaller vortex. For the large vortex, a low-level vorticity contour is chosen, since no stripping is observed (\( \omega_l / \omega_{\text{max}} = 0.169 \), contour #2 in Table I).

2. Quantification

After identifying the vortex cores, we quantify and classify them. In an inviscid flow, the circulation \( \Gamma \) and area \( A \) within every vorticity contour are constant, and we monitor these quantities for the nominated vorticity level within each vortex core along with the aspect ratio \( \lambda \) of the fitting ellipse. We also calculate the mean distance between the vortex centers. We define the vortex center as the vorticity-weighted mean position of a point within the vortex’s nominated vorticity level:

\[
\text{TABLE II. Run information. Note: NGP is the approximate diameter, in grid lengths, of the smaller vortex at } t = 0.
\]

\[\begin{array}{cccccccc}
\text{Run} & N^2 & \nu_2 & \nu_4 & \text{NGP} & \frac{\Gamma_f}{\Gamma_{\text{ref}}} & \frac{\epsilon_f}{\epsilon_{\text{ref}}} & \frac{\lambda_f}{\lambda_{\text{ref}}} \\
1 & 64^2 & 0 & 8.0 \times 10^{-6} & 6 & 0 & 0.8401 & 0.9885 \\
2 & 128^2 & 0 & 5.0 \times 10^{-7} & 12 & 0 & 0.9040 & 0.9986 \\
3 & 192^2 & 0 & 9.88 \times 10^{-8} & 18 & 0.4751 & 0.9439 & 0.9999 \\
4 & 256^2 & 0 & 3.125 \times 10^{-8} & 24 & 0.5014 & 0.9542 & 0.9999 \\
5 & 512^2 & 0 & 1.953 \times 10^{-9} & 48 & 0.5387 & 0.9632 & 0.9999 \\
6 & 64^2 & 8.132 \times 10^{-2} & 0 & 9 & 0 & 0.1077 & 0.2823 \\
7 & 128^2 & 2.048 \times 10^{-3} & 0 & 12 & 0 & 0.4165 & 0.6575 \\
8 & 192^2 & 9.10 \times 10^{-4} & 0 & 18 & 0 & 0.5842 & 0.8128 \\
9 & 256^2 & 5.12 \times 10^{-4} & 0 & 24 & 0 & 0.6819 & 0.8836 \\
10 & 512^2 & 1.28 \times 10^{-4} & 0 & 48 & 0 & 0.8308 & 0.9669 \\
11 & \text{CS} & \mu = 0.16 & & & 0.7203 & 0.9795 & 0.9836 \\
12 & \text{CS} & \mu = 0.08 & & & 0.7009 & 0.9869 & 0.9902 \\
13 & \text{CS} & \mu = 0.06 & & & 0.6951 & 0.9893 & 0.9924 \\
\end{array}\]
Normal Diffusion

\[ \text{Hyperdiffusion} \]

\[ (x_c, y_c) = \frac{\iiint (x, y) \omega \, dx \, dy}{\iiint \omega \, dx \, dy} \]

In the PS calculations, the resolution is generally too coarse for an accurate straightforward quantification of the smaller vortex. In the 64^2 calculation, for example, there are only six points across the smaller vortex initially. We therefore interpolate the data from our calculations onto a high-resolution grid (typically 1024^2 or 960^2) by setting the higher modes in the spectral space to zero. The subrange that just bounds the smaller vortex is isolated for quantification. In the CS calculations, we use the algorithm's inherent cubic interpolation for a sufficiently accurate determination of the diagnostics.

IV. RESULTS

A. Juxtaposition of the CS and PS computed fields

Figures 2 and 3 compare the contour-rendered vorticity fields of some of the calculations at two different times, \( t = 5 \) and 50. The contour levels shown correspond with the discrete levels in CS (see Table I). The most significant differences at early times develop on the smaller vortex. In the PS calculation, the outer vorticity levels have essentially collapsed onto the innermost contour—see Table III for the minimum distance across the innermost five contours in CS at various spatial resolutions. This level of gradient intensification is well beyond the reach of the PS method (one would require a resolution several times in excess of \( 2\pi/9.97 \times 10^{-5} \approx 63,000 \) in each direction).

In PS, the gradients reach their peak at about \( t = 5 \) and rapidly decrease thereafter because of the high level of localized dissipation. This "rebound" is the most significant event in the PS evolution. We will see that the failure to capture and maintain high vorticity gradients accounts for the rapid divergence of the two calculation methods after this time.

After the rebound, the smaller vortex in the PS calculation erodes as the shear of the larger vortex continues to strip peripheral vorticity. Uncharacteristic of inviscid dynamics, vorticity is continually replenished along the periphery of the smaller vortex by down-gradient smoothing. This combined action of stripping plus diffusion erodes the smaller vortex and causes its peak vorticity to decrease (though there can be a significant transient increase when hyperdiffusion is used; see Sec. IV C and Ref. 14). Eventually, the vortex is stretched into a thin filament when its peak vorticity falls below about 7.4 times the external shear. In the CS calculation, where there is no down-gradient dissipation, the sharp gradients remain and the peak vorticity stays constant. (Clearly, the "surgery" in CS is not acting like diffusion.)

As the smaller vortex erodes in the PS calculation, weak areas of circulation are deposited into a "halo" surrounding the larger vortex, leading to further differences in the long-time behavior of the PS and CS calculations. The effect of

FIG. 2. Vorticity contours at \( t = 5 \) for the PS and CS calculations. Cases (a)–(c) are the runs 1, 4, and 5; (e)–(g) are the runs 6, 9, and 10, while (d) and (h) are runs 11 and 12.
this halo on the larger vortex is seen in Fig. 4, which shows the larger-vortex aspect ratio $\lambda_1$ for the hPS and CS calculations. A good agreement in both magnitude and frequency is observed from $t=0$ to about $t=5$; thereafter, a monotonic decrease in aspect ratio (an increase in ellipticity) is observed in the lower-resolution PS calculations, until the vorticity within the filament ring becomes sufficiently weak to no longer have a significant, coordinated impact on the larger vortex (see Sec. IV C for differences between normal diffusion and hyperdiffusion).

For the smaller vortex, the aspect-ratio diagnostic ($\lambda_2$, Fig. 5) exhibits an irregular evolution in all but the highest resolution hPS calculation. The observed differences between the methods is a result of the rapid reduction of edge-vorticity gradients in the PS calculations just after $t = 5$ (the "rebound"). After the rebound, the structure of the smaller

<table>
<thead>
<tr>
<th>Time</th>
<th>$\mu = 0.16$</th>
<th>$\mu = 0.10$</th>
<th>$\mu = 0.05$</th>
<th>$\mu = 0.025$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$4.17 \times 10^{-2}$</td>
<td>$4.17 \times 10^{-2}$</td>
<td>$4.17 \times 10^{-2}$</td>
<td>$4.17 \times 10^{-2}$</td>
</tr>
<tr>
<td>2</td>
<td>$1.20 \times 10^{-2}$</td>
<td>$1.19 \times 10^{-2}$</td>
<td>$1.19 \times 10^{-2}$</td>
<td>$1.19 \times 10^{-2}$</td>
</tr>
<tr>
<td>4</td>
<td>$4.79 \times 10^{-4}$</td>
<td>$5.02 \times 10^{-4}$</td>
<td>$5.12 \times 10^{-4}$</td>
<td>$5.26 \times 10^{-4}$</td>
</tr>
<tr>
<td>5</td>
<td>$1.66 \times 10^{-5}$</td>
<td>$8.32 \times 10^{-5}$</td>
<td>$1.02 \times 10^{-4}$</td>
<td>$9.97 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

FIG. 3. The same as in Fig. 2 except at $t=50$.

vortex is significantly different in the two calculation methods. The patch-like structure in the CS calculation has different internal modes of oscillation than the more rounded structure seen in the PS calculations. Only the best-resolved PS calculation is able to reproduce the regular frequency of

FIG. 4. Aspect ratio evolution for the larger vortex (under hyperdiffusion and in CS).
of dissipation (the slope of the Z vs t curve) is maximal just after the rebound (around t = 5) in the hPS calculations, when the gradients are highest.

C. Hyperdiffusion versus normal diffusion

Next, we compare hyperdiffusion with normal diffusion. During the stripping period, in the nPS calculations the peak vorticity of the smaller vortex decreases monotonically, whereas in the hPS calculations it increases. Figure 7 shows the vorticity profiles of the 128^2 runs 2 (hPS) and 7 (nPS) (consult Table II) at t = 5 and 10. In the hPS case, the peak vorticity of the smaller vortex increases by a factor of 1.3 by t = 10.

To elucidate the competition between hyperdiffusion and resolution, we varied the resolution from 128^2 to 512^2, while keeping \( u_0 = 5 \times 10^{-7} \). Figure 8 shows the magnitude of "overshoot" for three different resolutions. The peak vorticity of the smaller vortex increases from 6.28 to about 8.30 and then decreases. The results prove that this unphysical "overshoot" occurs because of hyperdiffusion, and is not affected by temporal or spatial resolution. As a consequence of this numerical artifact, the lifetime of smaller vortices is extended (relative to normal diffusion), since the effect of the external strain is relatively weaker (see Ref. 14 for further details). Ultimately, when there are only a few grid points spanning a vortex (due to the continual erosion of the vortex), the process appears more diffusive and the peak vorticity decreases until vortex destruction.

In work to be published, we are presently examining the evolution of a single isolated high-gradient vortex suggested by Farge: \( \omega(r,0) \sim r^{-1/2} \). We observe a similar overshoot phenomenon. We also observe that both the magnitude and persistence of the overshoot increase with higher-degree hyperdiffusions.
Normal diffusion

Hyperdiffusion

FIG. 7. Vorticity profiles in hyperdiffusive and normal diffusive P3 calculations (120\(^2\) resolution).

FIG. 8. The effects of resolution on the hyperdiffusive "overshoot" phenomenon.

FIG. 9. The centroid distance between the two vortices as a function of time.
After the disappearance of the smaller vortex in the nPS calculations and the 64\(^2\) and 128\(^2\) hPS calculations, a further numerical artifact of hyperdiffusion is revealed. Notice that in the hPS calculations, the larger vortex becomes progressively more elliptical, while in the nPS calculations, it becomes more circular (see Fig. 10). It is because that with normal diffusion, the filamentary remnants of the smaller vortex rapidly weaken and diffuse and behave quasipassively around the larger vortex. However, with hyperdiffusion, the filamentary remnants do not diffuse as rapidly, and remain much stronger and localized, hence dynamically more active—see Fig. 11. This permits an interaction between the filament ring and the vortex, resulting in a curious instability that elongates the vortex. Ultimately, the elongation abates when the ring vorticity has diffused sufficiently.

Figure 12 shows the circulation within the smaller vortex core divided by its initial value, \(\Gamma_f/\Gamma_{s0}\), versus time. Note that much higher circulation is retained in the smaller vortex in the CS calculation than in any of the PS calculations. These figures clearly indicate that much greater resolution would be required in PS to achieve results comparable to what is observed in CS.

To see what impact the ring has in the CS calculations, Fig. 13 presents the evolution of the circulation contained in the filaments \(\Gamma_f\) divided by the initial circulation of the smaller vortex \(\Gamma_{s0}\), where

\[
\Gamma_f = \Gamma - \Gamma_x - \Gamma_i.
\]

At higher resolution (smaller \(\mu\)) much more of the initial filamentary debris is preserved through the calculation. Over a long period of time, the extra filamentary vorticity modifies the evolution of the larger vortex, causing it to become slightly more elliptical (Figs. 4 and 14). However, this effect is not as pronounced as in the hPS calculations, because much less filamentary vorticity is formed in the CS calculations (recall that the stripping has essentially stopped by \(t = 10\) in CS while it continues as a result of down gradient dissipation in PS).

V. CONCLUDING REMARKS

Consistent with previous results comparing the CS and PS methods,\(^9\) we find that just eight vorticity levels in CS are sufficient to produce an excellent early time comparison. For large-scale features, the comparison remains good for longer times. However, differences develop quickly at small scales, particularly in the case studied here, where, in the absence of dissipation, vorticity gradients develop that are well beyond the currently used resolution limits of the PS method (or of any conventional, grid-based method). In our PS calcula-
tions, the smaller vortex not only falls far short of reaching the level of gradient intensification observed in the inviscid CS calculation, but also erodes when the low level vorticity constantly being regenerated along its periphery is stripped away by the adverse shear of the larger vortex. This erosion is accompanied by a rounding off of the top of the vortex and a reduction in its peak vorticity, culminating in the shearing out of the vortex when the applied strain rate to peak vorticity reaches a critical value.

The short lifetime of small vortices at low resolution in conventional models implies that both the partial merger (PM) and the partial straining-out (PSO) regimes of inelastic vortex interactions are suppressed in many of the contemporary simulations of 2-D turbulence. These regimes are the only ones in which the product of the interaction of two vortices is two vortices, the smaller of the two having decreased in size. These regimes are also favored in a dilute gas of vortices, where many of the interactions will be grazing or weak ones. Such a situation is, anyway, difficult to study using conventional models, since only a small part of the domain would be occupied by the vortices.

For example, in the hyperdiffusive-PS calculation of 2-D turbulence presented in Ref. 2, no vortex is ever spanned by more than 25 grid points throughout the entire evolution, and most vortices are spanned by ten or fewer grid points. At early times, the situation is much worse. One may therefore expect, given the findings of the present paper, that the population characteristics and general structure of the vortices in this PS experiment are neither representative of an inviscid fluid (as Ref. 16 claims) nor of a normal viscous one. The "universal scaling theory" for 2-D turbulence, developed in Ref. 16, appears rather to only apply to 2-D turbulence simulations with a sufficiently restricted range of spatial scales (then, a narrow distribution of vortex sizes can remain narrow). This, we believe, is the main cause of the discrepancy between the theory and results of Refs. 2 and 16 and the CS results for 2-D turbulence presented in Ref. 17. Indeed, as pointed out in Ref. 17, the CS results for 2-D turbulence are much closer to the much higher resolution PS experiment of Ref. 18. Reference 18 achieved markedly higher resolution than Ref. 2 by using both a finer overall grid resolution and a larger-scale initial disturbance. We still lack a theory that can explain the results of 2-D turbulence simulations having a wide range of spatial scales.

As a final remark, our study has some practical implications for the ability to model the atmosphere and oceans. Present numerical models rely on some kind of artificial diffusion or subgrid-scale model to maintain smooth fields for stability. Such devices are not considered robust, i.e., as a good approximation of the true dissipative mechanisms; rather, they are there essentially for numerical stability. There is little hope, even with massive increases in computer power, to bridge the gap between model grid scales and the true physical dissipation scale. A statement made a decade ago by the atmospheric scientists McIntyre and Palmer (p. 832) appropriately describes the present status of numerical modeling of the atmosphere and oceans: "The question

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig13}
\caption{Here $\Gamma_t/\Gamma_{t_0}$ versus time in three CS calculations differing only in spatial resolution $\mu$.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig14}
\caption{Vorticity contours at $t=50$ for the three CS calculations.}
\end{figure}
then becomes how to choose the form of these (artificial dissipation) terms so as to do the least damage to the basic processes being modeled, including the quasi-two-dimensional fluid-dynamical irreversibility." Is it not now timely to develop hybrid numerical methods that do not rely on high down-gradient dissipation for stability?

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