

My research is centred around the interface between fundamental atmospheric–oceanic fluid dynamics and numerical modelling. Interplay between dynamics and numerics is thought to be essential to construct “accurate” and “efficient” numerical algorithms for geophysical flows in general and for global models of the atmosphere and oceans used in numerical weather and climate prediction, in particular. A major difficulty faced is the handling of advective nonlinearity coupling a vast range of spatio-temporal scales. Further, in the absence of suitable analytical solutions for complex, nonlinear flows, accuracy analysis of numerical algorithms proves challenging. To face this challenge, I have endeavoured to bridge the gap between the traditional tools usually used by geophysical modellers, like dispersion/dissipation properties of numerical algorithms for linear problems, and the error analysis usually carried out by numerical analysts using functional analysis techniques, though again mostly for linear equations.

To this end, I have worked to fuse the numerics and dynamics as far as possible. Specifically I have put forward and demonstrated the usefulness of “balance” in the design and accuracy analysis of numerical algorithms for the single-layer shallow-water equations [2,3,10,11] and multi-layer primitive-equation models [5]. An important subset of atmospheric–oceanic flows is believed to be largely balanced. Broadly speaking, balance means that some diagnostic functional relations, called balance relations or balance conditions, hold between the mass (pressure) and velocity fields in the continuous limit. I have extensively worked on the representation of balance in discrete models based on novel contour-advective semi-Lagrangian (CASL) algorithms [1,2,4] as well as the conventional pseudo-spectral and semi-Lagrangian algorithms. This fundamental contribution has been recognised by NERC and EPSRC by awarding a three-year fellowship under the thematic programme of Environmental Mathematics and Statistics (EMS). The fellowship can make a major contribution to numerical modelling of global circulation, weather and climate in two inter-related ways: (i) a hybrid approach, i.e. a Lagrangian for vortical flow and an Eulerian for wave fields, (ii) the use of balance in analysing the accuracy of numerical algorithms.

My other major contributions so far are briefly outlined here. For the  $f$ -plane shallow-water equations, I have designed algorithms to explore hierarchies of balance relations up to arbitrarily high orders in Froude/Rossby numbers. This work [3] has established the asymptotic nature of such balance relations. In [4] I have revealed an inherent indeterminacy in the use of asymptotic expansions to find balance relations used to invert potential vorticity (PV). I have compared various methods to invert PV using Rossby-number expansion relations and among other things I have shown a noticeable failure of the method introduced by Warn *et al.* (*Q. J. Roy. Meteorol. Soc.*, **121**, 723–739). In collaboration with V. Esfahanian and S. Ghader, I have worked on compact and super compact finite differencing schemes as applied to various linear and nonlinear equations [6,9,12]. In collaboration with Michael McIntyre [8,9], we have addressed the long standing question of local mass conservation in PV-based balanced models. We found that the high-order truncation [3] PV-based balanced can be made locally mass conserving at arbitrary order, leading to a new set of balance equations called hyperbalance equations. The hyperbalance PV-based balanced models have the theoretically appealing properties of (i) removing a small, yet important problem with

simultaneous conservation of PV, circulation and mean mass in PV-based balanced models [4], and (ii) globally conserving all the moments of PV (analogues of Casimirs in Hamiltonian balanced models). In a systematic study of numerical accuracy of high-order truncation and hyperbalance PV-based balance models, for the  $f$ -plane shallow-water equations we have found no systematic trade-off between local mass conservation and numerical accuracy in hyperbalance PBMs. That is, McIntyre–Norton conjecture (*J. Atmos. Sci.*, **57**, 1214–1235) is not supported in  $f$ -plane shallow-water equations where spontaneous-adjustment emission of gravity waves is weak.

Publications, Articles in Journals:

1. Dritschel, D.G., Polvani, L.M. and A.R. Mohebalhojeh, 1999: The contour-advective semi-Lagrangian algorithm for the shallow water equations. *Mon. Wea. Rev.*, **127**, 1551–1565.
2. Mohebalhojeh, A.R. and D.G. Dritschel, 2000: On the representation of gravity waves in numerical models of the shallow water equations. *Q. J. Roy. Meteorol. Soc.*, **126**, 669–688.
3. Mohebalhojeh, A.R. and D.G. Dritschel, 2001: Hierarchies of balance conditions for the  $f$ -plane shallow-water equations. *J. Atmos. Sci.*, **58**, 2411–2426.
4. Mohebalhojeh, A.R., 2002: On shallow-water potential vorticity inversion by Rossby-number expansions. *Q. J. Roy. Meteorol. Soc.*, **128**, 679–694.
5. Mohebalhojeh, A.R. and D.G. Dritschel, 2004: Contour-advective semi-Lagrangian algorithms for many-layer primitive equation models. *Q. J. Roy. Meteorol. Soc.*, **130**, 347–364.
6. Esfahanian, V., Ghader, S., and A.R. Mohebalhojeh, 2005: On the use of the super compact schemes for spatial differencing in numerical models of the atmosphere. *Q. J. Roy. Meteorol. Soc.*, **131**, 2109–2129.
7. Mohebalhojeh, A.R. and M.E. McIntyre, 2006: Local mass conservation and velocity splitting in PV-based balanced models. Part I: The hyperbalance equations. *J. Atmos. Sci.*, in press.
8. Mohebalhojeh, A.R. and M.E. McIntyre, 2006: Local mass conservation and velocity splitting in PV-based balanced models. Part II: Numerical results. *J. Atmos. Sci.*, in press.
9. Mohebalhojeh, A.R. and D.G. Dritschel, 2006: Assessing the numerical accuracy of complex spherical shallow water flows. *Mon. Wea. Rev.*, in press.

Publications, Conference Contributions:

1. Dritschel, D.G. and A.R. Mohebalhojeh, 2000: The contour-advective semi-Lagrangian algorithm: Keeping the balance. in proceeding of the European Centre for Medium-Range Weather Forecasts (ECMWF) Workshop on *Developments in numerical methods for very high resolution global models*, 5–7 June 2000, 119–136.
2. Mohebalhojeh, A.R., 2002: Waves and vortices in numerical models of the atmosphere and oceans. In proceeding of the 9<sup>th</sup> Asian Congress of Fluid Mechanics, 27–31 May 2002, IUT, Isfahan, Iran.
3. Esfahanian, V., Ghader, S., and A.R. Mohebalhojeh, 2004: Super compact spatial differencing for the linear and nonlinear geophysical fluid dynamics problems. In proceedings of the Third International Conference on Computational Fluid Dynamics, ICCFD3, Toronto, 12–16 July 2004, 423–428.